

Ex. the non-degenerating form follows "up to" transformation, reflection, and rotation.

Name
Ellipsoid

Equation
$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

~~cone~~ Cone

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 0$$

Elliptic Paraboloid

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z}{c} = 0$$

Ex. $x^2 - y^2 - z^2 - 4x - 2z + 3 = 0$

① Rewrite

$$0 = x^2 - y^2 - z^2 - 4x - 2z + 3$$

$$0 = (x^2 - 4x + 4) + (-y^2) + (-z^2 - 2z - 1)$$

$$0 = (x-2)^2 - y^2 - (z+1)^2$$

Let's understand the cross-section of this picture with respect to (wrt) the coordinate planes (shifted)

when $z = k$ (constant)

i.e. $(x-2)^2 - y^2 = (k+1)^2$ | hyperbola

when $y = k$

i.e. $(x-2)^2 - (z+1)^2 = k^2$ | hyperbola

when $x = k$

$y^2 + (z+1)^2 = (k-2)^2$ | ellipse

Conic Sections

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

ellipse

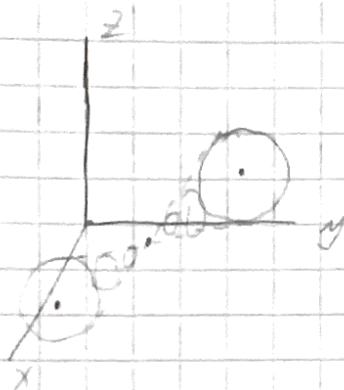
$$\frac{x^2}{a^2} - \frac{y}{b} = c$$

parabola

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

hyperbola

Picture:



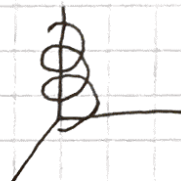
Section 13.1 Space Curves

A space curve is a function

$$\vec{r}: I \rightarrow \mathbb{R}^3.$$

Ex. The Helix is the space curve

$$\vec{r}(t) = \langle \cos(t), \sin(t), t \rangle \Rightarrow$$



shadow in x-y plane



Definition: Limit of a space curve

$$\vec{r}(t) = \langle x(t), y(t), z(t) \rangle$$

at $t=a$ is the componentwise limit if they all exist.

$$\lim_{t \rightarrow a} \vec{r}(t) = \lim_{t \rightarrow a} \langle x(t), y(t), z(t) \rangle = \langle \lim_{t \rightarrow a} x(t), \lim_{t \rightarrow a} y(t), \lim_{t \rightarrow a} z(t) \rangle$$

Ex. Compute the limit of $\vec{r}(t) = \langle 4 + \sin(20t)\cos(t), [4 + \sin(20t)]\cos(t), 4 + \sin(20t) \rangle$ at $t = \frac{5\pi}{8}$.

$$F: I \rightarrow \mathbb{R}^n$$

The limit of $F(t)$ is completely computed wise